Mixed Exercise

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

12 Expand and simplify if possible:

a $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$

13 Rationalise the denominator and simplify:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2} - 1}$ **c** $\frac{3}{\sqrt{3} - 2}$ **d** $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$ **e** $\frac{1}{(2 + \sqrt{3})^2}$ **f** $\frac{1}{(4 - \sqrt{7})^2}$

14 **a** Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b Hence, fully factorise $x^3 - x^2 - 17x - 15$.

(E) **15** Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

(E/P) **16** Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**

(E) **17** Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$. **(2 marks)**

(E) **18** Factorise completely $x - 64x^3$. **(3 marks)**

(E/P) **19** Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

(E/P) **20** Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

(P) **21** A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

(E) **22** Show that $\frac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

(E/P) **23** Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

(E/P) **24** Given that $\frac{4x^3 + x^{\frac{1}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b. **(2 marks)**

Answers

11 **a** 21 877

b $(5x + 6)(7x - 8)$

When $x = 25$, $5x + 6 = 131$ and $7x - 8 = 167$; both 131 and 167 are prime numbers.

12 **a** $3\sqrt{2} + \sqrt{10}$ **b** $10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{15}$

c $24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{14}$

13 **a** $\frac{\sqrt{3}}{3}$ **b** $\sqrt{2} + 1$ **c** $-3\sqrt{3} - 6$

d $\frac{30 - \sqrt{851}}{-7}$ **e** $7 - 4\sqrt{3}$ **f** $\frac{23 + 8\sqrt{7}}{81}$

14 **a** $b = -4$ and $c = -5$ **b** $(x + 3)(x - 5)(x + 1)$

15 **a** $\frac{1}{4}x$ **b** $256x^{-3}$

16 $\frac{5}{\sqrt{75} - \sqrt{50}} = \frac{1}{\sqrt{3} - \sqrt{2}} = \sqrt{3} + \sqrt{2}$

17 $-36 + 10\sqrt{11}$

18 $x(1 + 8x)(1 - 8x)$

19 $y = 6x + 3$

20 $4\sqrt{3}$

21 $3 - \sqrt{3}$ cm

22 $\frac{4 - 4x^{\frac{1}{2}} + x^1}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$

23 $\frac{11}{2}$

24 $4x^{\frac{5}{2}} + x^2$, $a = \frac{5}{2}$ $b = 2$