Further Maths Extension

1) *[MAT 2002 1B]*

Of the following three alleged algebraic identities, at least one is wrong.

(i) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(x-z)(y-x)$
(ii) $yz(z-y)+zx(x-z)+xy(y-x)$
$=(z-y)(z-x)(y-x)$
(iii) $yz(x+y)+zx(z+x)+xy(y+x)$
$=(z+y)(z+x)(y+x)$

Which of the following statements are correct? Tick all that apply.

- (i)
- (ii)
- (iii)

2) *[MAT 2007 1E]*

If x and n are integers then

$$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

is:

- negative when $n>5$ and $x<5$
- negative when n is odd and $x>5$
- negative when n is a multiple of 3 and $x>5$
- negative when n is even and $x<5$

3) *[MAT 2007 1A]*

Let r and s be integers. Then

$$\frac{6^{r+s}\times 12^{r-s}}{8^r\times 9^{r+2s}}$$

is an integer if

- $r+s\leq 0$
- $s\leq 0$
- $r\leq 0$
- $r\geq s$

4) [SMC 2014 Q24] Which of the following is smallest?

- $10-3\sqrt{11}$
- $8-3\sqrt{7}$
- $5-2\sqrt{6}$
- $9-4\sqrt{5}$
- $7-4\sqrt{3}$

Further Maths Extension

[MAT 2002 1B]

Of the following three alleged algebraic identities, at least one is wrong.

(i) $yz(z-y)+zx(x-z)+xy(y-x)$
$= (z-y)(x-z)(y-x)$

(ii) $yz(z-y)+zx(x-z)+xy(y-x)$
$= (z-y)(z-x)(y-x)$

(iii) $yz(x+y)+zx(z+x)+xy(y+x)$
$= (z+y)(z+x)(y+x)$

Which of the following statements are correct? Tick all that apply.

- (i)
- (ii)
- (iii)

Solution: (ii) only

[MAT 2007 1E]

If x and n are integers then

$$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

is:

- negative when $n>5$ and $x<5$
- negative when n is odd and $x>5$
- negative when n is a multiple of 3 and $x>5$
- negative when n is even and $x<5$

Solution: n is odd and $x > 5$

[MAT 2007 1A]

Let r and s be integers. Then

$$\frac{6^{r+s}\times 12^{r-s}}{8^r\times 9^{r+2s}}$$

is an integer if

- $r+s\leq 0$
- $s\leq 0$
- $r\leq 0$
- $r\geq s$

$$= \frac{2^{r+s}\times 3^{r+s}\times 2^{2r-2s}\times 3^{r-s}}{2^{3r}\times 3^{2r+4s}}$$
$$= 2^{-s}\times 3^{-4s}$$

This is an integer only if $s \leq 0$.

[SMC 2014 Q24] Which of the following is smallest?

- $10-3\sqrt{11}$
- $8-3\sqrt{7}$
- $5-2\sqrt{6}$
- $9-4\sqrt{5}$
- $7-4\sqrt{3}$

Note that:

$$10-3\sqrt{11}=\sqrt{100}-\sqrt{99}$$

The other options can similarly be written as $\sqrt{n+1}-\sqrt{n}$. The greater the n, the smaller the number, so the answer is $10-3\sqrt{11}$.